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Multiple transverse fracture in 90° cross-ply laminates of a glass fibre-reinforced polyester

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Specimens of a 90° cross-ply glass-reinforced polyester were tested in tension in a direction parallel to one of the directions of reinforcement. Extensive cracking of the transverse ply occurred at strains much lower than the resin failure strain. These cracks formed in a direction parallel to the transverse reinforcement and showed a remarkably even crack spacing. Results of crack spacing measurements are presented against applied stress for specimens with differing transverse-ply thicknesses. The transverse-crack spacing was found to decrease with increasing applied stress and to increase with increasing transverse-ply thickness. There was no evidence of debonding between the plies during cracking and a multiple cracking theory in which the plies remain elastically bonded has been presented which can account for the results.

1. Introduction

The high mechanical performance of fibre-reinforced plastics is generally governed by the fibre properties. However, the matrix properties can be all important under certain load conditions and fibre configurations. Maxtix microdamage has been found to occur in cross-ply fibre-reinforced plastics even when the matrix has a higher failure strain than the fibres [1–7]. Furthermore, the strain at which this damage occurs is much less than the matrix failure strain. Although the structural integrity of the composite is maintained, and further loading is possible, the material may have to be considered to have failed if it is in use as a pressure or liquid container, as these matrix cracks may result in a fluid leak path. Thus the ultimate strength of the GRP may not be realized under these conditions without recourse to a lining material.

It has been found that this low-strain matrix damage is generally associated with fibres that lie at right angles to the direction of the main applied stress [1–4]. Hence, unidirectional continuous fibre structures incur little damage when the applied stress is in the fibre direction whilst the cross-ply structures experience large amounts of damage in the transverse plies.

Generally, the onset of matrix microfailure in glass-reinforced polyester samples occurs between 0.2 and 0.5% strain and is associated with the characteristic "knee" found in the stress–strain curves of cross-ply laminates [2, 8]. Kies [9] has treated this low-strain microdamage phenomenon theoretically. He calculated the localized microstrains between two fibres in a resin when a uniaxial tension is applied in a direction perpendicular to the fibres and found that a strain magnification occurred between them. This strain magnification increase as the interfibre spacing decreases and typically approaches 20 as the interfibre distance tends to zero. This type of analysis suggests that to develop the full strength of a cross-ply laminate without damaging the matrix requires resins with failure strains in excess of 20%. Hermann and Pister [10] and Schulz [11] have extended the analysis to include biaxial strains. In agreement with these theoretical analyses, it has been reported that the change in mechanical properties due to resin cracking occurs at higher stress levels when resins which have a higher elongation at break are used [12], and that by far the greatest number of cracks started in regions of high fibre density, especially when the fibres were nearly in contact or were in contact [4].

An alternative approach to the microfailure phenomenon can be elucidated from the work of Niederstadt [13], Broutman [14] and Puck [15] who infer that failure of a glass-reinforced resin structure starts with debonding at the glass–resin interface of transverse fibres. Once the debonding starts, less cross-sectional area will be available for a proper stress distribution, causing stress concentrations and hence further debonding.

If the matrix has a lower failure strain than the fibres, matrix damage will occur in unidirectional composites when stressed in the fibre direction. Under these conditions multiple transverse matrix fracture can occur. This phenomenon has been investigated theoretically by Aveston *et al.* [16] for unbonded fibres, and by Aveston and Kelly [17] for bonded fibres. Experimental evidence for multiple fracture has been presented for glass-reinforced cement [18], for gypsum reinforced with PVC or glass [19, 20], and epoxy resin reinforced by steel wire [21]. A recent review of multiple cracking in brittle matrix composites has been presented by Aveston *et al.* [22].

In this paper we report a systematic investigation of transverse cracking in cross-ply structures and the results of an investigation of the effects of transverse-ply thickness and applied stress on the resulting behaviour.

2. Experimental procedure

Experiments were conducted on composites made from Crystic 390, a polyester resin supplied by Scott Bader Ltd, reinforced with a unidirectional roving cloth Tyglas Y119 of weight 745 $g\,m^{-2}$, manufactured by Fothergill and Harvey Ltd. The glass cloth has a weft which comprises 10% of the total cloth weight. In order to have unidirectional plies in the samples, this weft was removed from the middle of the glass fabric, so that test specimens could be cut from totally unidirectional material. Keeping the weft at the edges of the cloth ensured that the alignment of the fibres was maintained during the lay-up procedure when slight tension was applied to the warp. Symmetrical cross-ply specimens were made as illustrated in Fig. 1, with the transverse ply sandwiched between the longitudinal plies.

One layer of cloth was used for each of the longitudinal plies. The thickness of the transverse ply was varied by using different numbers of cloth layers. Slight tension was maintained on the fibres during lay-up and cure. The specimens were squeezed to eliminate any excess resin and were gelled at room temperature for 24 h and post-cured for 4 h at 80° C. Specimens were made with transverse-ply thicknesses ($2d$) between 0.3 and 4 mm.

A set of about eight specimens of dimensions 20 mm x 220 mm were cut from each laminate. They were cut so that their length was parallel to the fibre direction in the longitudinal (outer) plies. The specimens were tensile tested on an "Instron" machine at a cross-head speed of 0.05 $cm\,min^{-1}$. Each of the specimens cut from the same laminate was loaded to a different strain level from that at which initial damage in the transverse ply occurs, increasing in increments of 0.2% to its ultimate failure strain (about 1.8%). In this way a complete picture of damage versus applied stress and strain could be obtained. The strain was measured by strain gauges mounted directly on the specimen. Additional information was supplied by an acoustic emission transducer fixed to each specimen. Time lapse photography was also employed on some specimens to observe the cracking sequence.

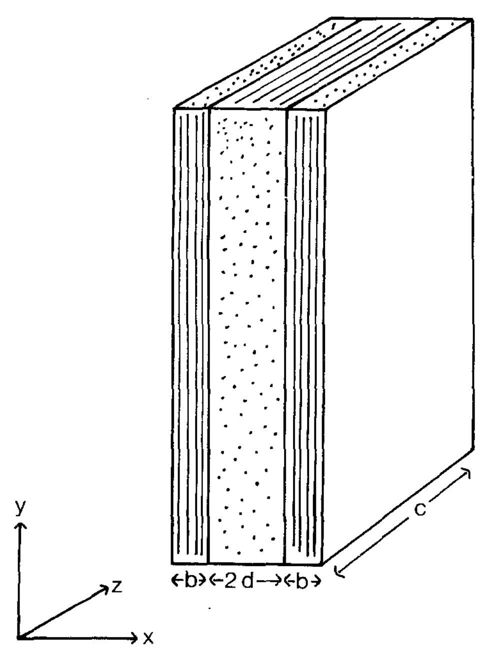

Figure 1 Specimen model.

The effect of the transverse cracks on the structural integrity of the specimens was investigated by measuring the transverse tensile strength, i.e. along the x-direction in Fig. 1, before and after the normal tensile testing of the specimens.

Testing in the x-direction was achieved by cutting a 20 mm length from a normal test specimen, resulting in a 20 mm square specimen in the y–z plane. 20 mm square metal blocks were cemented to the two large faces of the specimen and these were pinned to the Instron cross-head and load cell attachment with specially designed fittings which permitted rotational movement in the vertical plane. Tensile tests in which the bond between the metal block and the specimen broke were disregarded.

The interlaminar shear strength of the ply interfaces was also measured by direct shear experiments using an offset flat lap joint type of specimen.

3. Experimental results

Initial tensile experiments on 90° cross-ply laminates showed transverse cracks developing above strains of about 0.4%. These cracks occurred with a remarkably even crack spacing at strains greater than about 0.8%. The spacing between the cracks was found to depend on the thickness of the transverse ply. For a given strain it was found that the crack spacing increased as the transverse-ply thickness was increased. From these early observations a systematic investigation was begun.

Examples of the transverse cracking are shown in Fig. 2. the specimens have transverse-ply thicknesses of 0.75, 1.5 and 2.6 mm and have been strained to 1.6%, i.e. to just below the failure strain of the outer plies. The even crack spacing and its dependence on ply thickness is clearly shown.

The cracks in the specimens occur in the transverse ply. They form in a direction perpendicular to the applied stress and parallel to the transverse fibres and generally extend the full width of the specimen. The cracks are generally associated with voids and areas of high fibre volume fraction but cracks across large resin-rich areas have also been observed. Crack branching is fairly common and examples can be seen in Fig. 2. Microstructural investigations indicate that the cracks penetrate a little way into the outer plies and are tied by the longitudinal fibres as shown in Fig. 3. This effect is more readily seen for the specimens with the

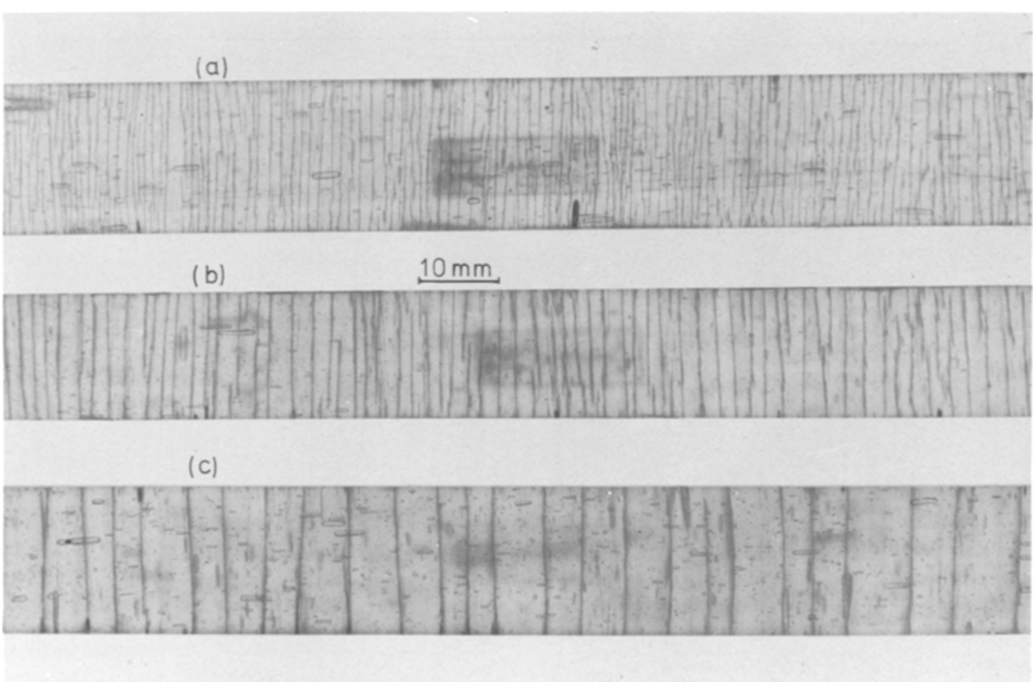

Figure 2 Transverse cracking in specimens with transverse-ply thickness of (a) 0.75 mm, (b) 1.5 mm and (c) 2.6 mm, strained to 1.6%.

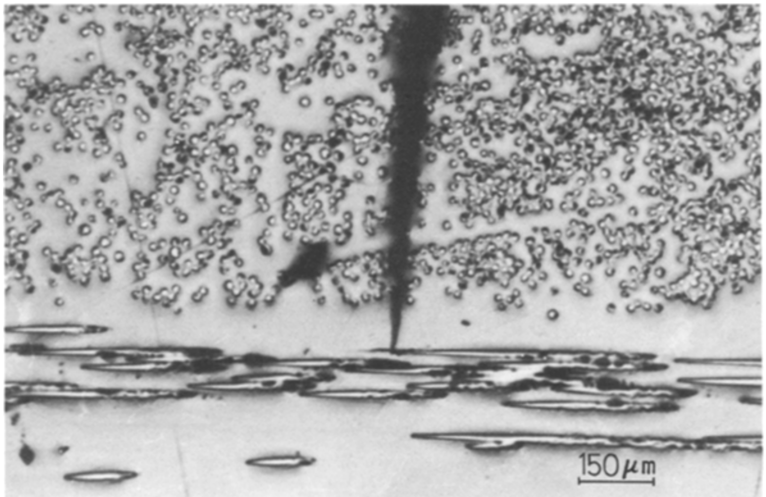

Figure 3 Interaction of a transverse crack with the longitudinal ply for a specimen with a transverse-ply thickness of 2.6 mm.

thicker transverse plies, where penetrations of about 0.1 mm were observed for a ply thickness of 3.2 mm. Debonding was rarely observed at the junction between the ply interfaces and the transverse cracks.

At approximately 1.6% strain, cracks running at ± 45° to the main transverse cracks were observed to form, and specimen failure occured soon after at strains around 1.8%. At failure, extensive debonding took place between the longitudinal and transverse plies.

Crack spacing was measured using a travelling microscope and an arithmetic mean was calculated for each specimen. The results of these measurements as a function of applied stress are shown in Fig. 4 for five transverse-ply thicknesses.

After the initial cracks occur, a sharp decrease in crack spacing results as the applied stress is

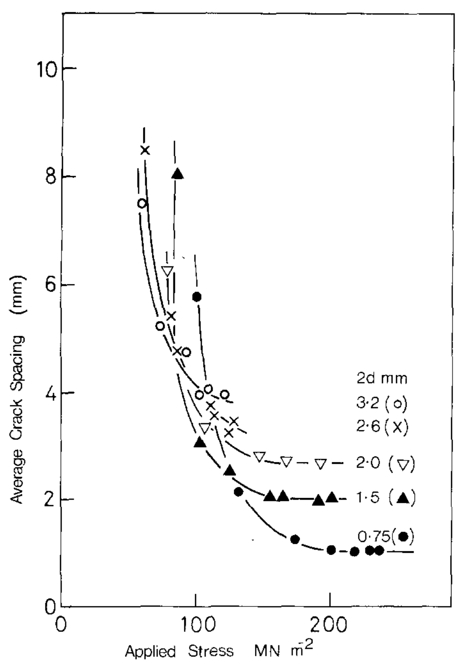

Figure 4 Average crack spacing as a function of applied stress for specimens with different transverse-ply thicknesses.

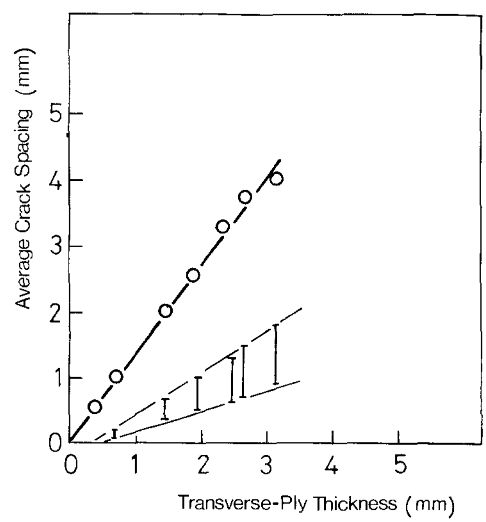

Figure 5 Average crack spacing as a function of transverse-ply thickness. The upper curve is the experimental results and the crack spacing values shown are taken from the apparent limiting value in Fig. 4. The lower curves are the upper and lower bounds of the theoretical crack spacing calculated from the theory of Section 4.1 at $\Delta\sigma_0 = 250\ \mathrm{MN\,m^{-2}}$.

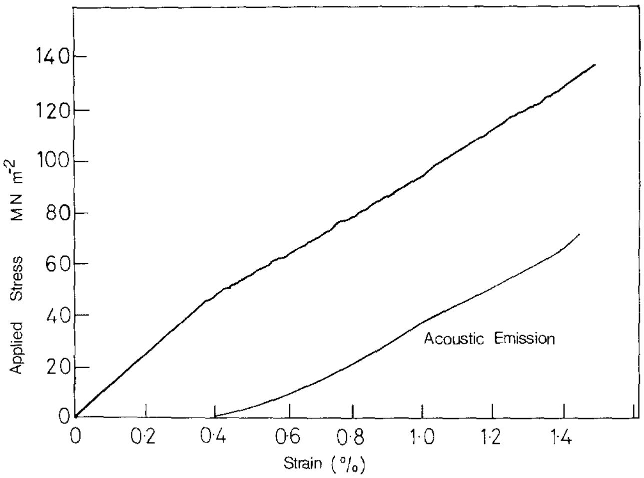

Figure 6 Typical stress–strain curve with "knee" at a strain of 0.4%. The lower curve is the integrated acoustic emission (arbitrary units).

TABLE I

Transverse-ply thickness (mm)	Initial cracking strain
0.75	0.48%
1.5	0.50%
2.0	0.44%
2.6	0.38%
2.7	0.40%
3.2	0.37%

increased. The curves eventually flatten out at higher applied stresses and the average crack spacing appears to reach a limiting value. This limiting value depends on the transverse-ply thickness as shown in Fig. 5.

The strain levels at which the initial cracking occurred are shown in Table I. These values are to be compared with a measured failure strain of 4.0% for the resin. The onset of cracking was detected in three ways, visually, by acoustic emission, and deduced from the position of the change of slope, the "knee", in the stress–strain curves. The values quoted in the table are the average of these three methods. The variation of the failure strain of the transverse ply from specimen to specimen is not fully understood and is the subject of further investigation.

Fig. 6 shows a typical stress–strain curve. The "knee" is clearly indicated at a strain of about 0.4%. The acoustic emission data in the figure indicate the correlation between the knee in the curve and the onset of cracking deduced from the acoustic counts.

The resin stress–strain curve is shown in Fig. 7. A cross-head speed of 0.05 cm min^{-1} was used during testing and a failure strain of 4.0% was measured.

The transverse tensile tests gave an average value of the bond strength of the longitudinal and transverse plies of 18 ± 3 MN m^{-2}. No significant difference was found between the bond strengths measured before and after the normal tensile testing of the specimens.

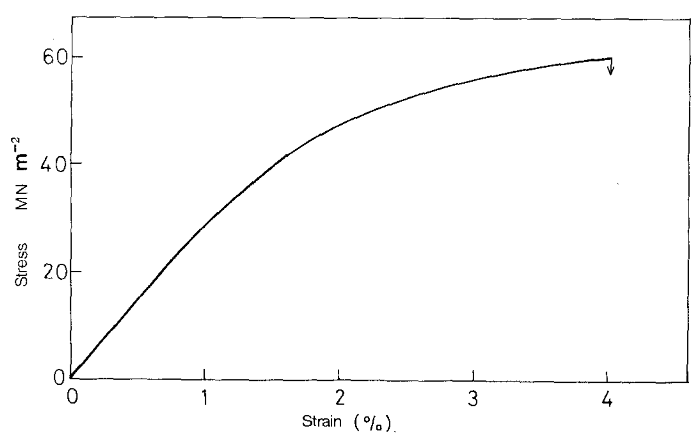

Figure 7 Stress–strain curve for the polyester resin. Failure strain = 4.0%.

The direct shear experiments produced an average value of $13 \pm 3\,\mathrm{MN\,m^{-2}}$ for the interlaminar shear strength of the ply interfaces.

4. Theory

4.1. Crack spacing

Multiple matrix cracking is observed in unidirectional composites when the matrix failure strain, ϵ_{mu}, is less than the fibre failure strain, ϵ_{fu}. In the case of the cross-ply laminates under investigation, ϵ_{mu} is greater than ϵ_{fu}. However, as low strain damage occurs in the transverse ply, the transverse ply acts as a material of low failure strain sandwiched between plies of a higher failure strain. Under these conditions, multiple transverse cracking can occur in the transverse ply.

Fig. 1 indicates the model used in the following theoretical investigation. Let us assume that the transverse ply has a unique breaking strain, ϵ_{tu}, and strength σ_{tu}. If a stress is applied in a direction parallel to the longitudinal plies, the transverse ply will fail at a stress σ_{tu}. The load carried by the transverse ply is then thrown onto the longitudinal plies and multiple cracking will occur in the transverse ply if the following inequality is satisfied:

$$\sigma_{lu} b \geqslant \sigma_{tu} d + \sigma_l' b, \qquad (1)$$

where σ_l' is the stress on the longitudinal plies when the transverse ply fails, and σ_{lu} is the strength of the longitudinal plies. If the inequality is not satisfied the longitudinal plies cannot withstand the extra load thrown onto them and single fracture results.

If multiple transverse cracking occurs, the equation governing the load transfer between longitudinal and transverse plies will be

$$\frac{\mathrm{d}F}{\mathrm{d}y} = 2c\tau_i \qquad (2)$$

where $\mathrm{d}F$ is the load transferred from the two longitudinal plies in distance $\mathrm{d}y$ at an interface shear stress τ_i. The behaviour of τ_i along the interface is of primary importantce in determining the rate at which the load is transferred back into the transverse ply and hence the resulting crack spacing. Aveston *et al.* [16], in their theoretical analysis of multiple cracking in unidirectional fibre reinforced composites, considered τ_i to be a constant along the fibre interfaces. The fibres are considered to be unbonded and, provided that a limiting shear stress is exceeded, the fibres and matrix can slide independently of each other. In the case of cross-ply laminates, the idea of a constant frictional shear stress at the ply interface is more difficult to conceive. If, however, τ_i can be considered as a constant equal to τ_i', multiple cracks will occur in the transverse ply spaced between y' and $2y'$ apart where y' is given by

$$y' = \frac{\sigma_{tu} d}{\tau_i'} \qquad (3)$$

This result is obtained by integrating Equation 2 and setting $F = 2\sigma_{tu} dc$. F refers to the total load transferred into the transverse ply (from two surfaces).

However, if the interface between the plies is considered to be elastically bonded, τ_i will be a function of y. An approximate elastic solution will be developed below to predict the average crack spacing as a function of applied stress for a sample whose transverse ply has a unique value of failure strain.

After the first crack has occurred in the transverse ply at a strain ϵ_{tu}, an additional stress, $\Delta\sigma$, is placed on the longitudinal plies. This additional stress has its maximum value $\Delta\sigma_0$ in the plane of the crack and decays with distance y from the crack surface as some load is transferred back into the transverse ply. Assuming an even load distribution in the longitudinal plies, we have

$$\Delta\sigma_0 = \sigma_a . \frac{b+d}{b} - E_1 \epsilon_{tu}, \qquad (4)$$

where σ_a is the applied stress on the sample and E_1 is the Young's modulus of the longitudinal ply. An approximate solution to the problem can be determined using a modified shear lag analysis. This is contained in the Appendix and it is found that in the case of cross-ply laminates

$$\Delta\sigma = \Delta\sigma_0 \exp(-\phi^{1/2} y), \qquad (5)$$

where

$$\phi = \frac{E_c G_t}{E_1 E_t} \left(\frac{b+d}{bd^2} \right), \qquad (6)$$

E_c is the Young's modulus of the composite in the y-direction and G_t is the shear modulus of the transverse ply in the y-direction.

From a simple force balance we have

$$\tau_i = -b \frac{d\Delta\sigma}{\mathrm{d}y} \qquad (7)$$

and so the shear-stress at the ply interface found by substituting equation 5 into Equation 7 and differentiating, is given by

$$\tau_i = b\Delta\sigma\phi^{1/2}\exp(-\phi^{1/2}y). \quad (8)$$

From Equations 2 and 8 the load F transferred back into the transverse ply at a given distance l from the plane of the crack will be

$$F = 2bc\,\Delta\sigma_0\,[1-\exp(-\phi^{1/2}l]. \quad (9)$$

The first crack in the transverse ply occurs when the load carried by it is equal to $2\sigma_{tu}dc$. This load is then transferred onto the longitudinal plies. Another crack can only occur when the transverse ply is again loaded to $2\sigma_{tu}dc$ by shear load transfer. If the value of σ_a is just the value at which the ply first breaks, the ply will only be loaded to this value again at infinity. For another crack to occur, σ_a (hence $\Delta\sigma_0$) must be increased to such value that l in Equation 9 lies within the length from the first crack to the nearest end of the specimen for F equal to $2\sigma_{tu}dc$.

If we initially assume that the first crack occurs in the middle of a specimen of length s, the following cracking sequence will occur.

(a) Initial crack at $\Delta\sigma_0 = \sigma_{tu}d/b$ and $\sigma_a = E_c\epsilon_{tu}$.

(b) Second and third cracks occur simultaneously at the ends of the specimen when the applied stress is such that

$$\Delta\sigma_0 = \sigma_{tu}\frac{d}{b}[1-\exp(-\phi^{1/2}s/2]^{-1}. \quad (10)$$

This result is obtained by substituting $s/2$ for l and $F = 2\sigma_{tu}dc$ in Equation 9. The crack spacing is $s/2$.

(c) The next series of cracks will occur midway between the present cracks, as the shear stress will build up from both cracks but will be of different signs. If the crack spacing is t, the total shear stress between two cracks will be

$$\tau = b\,\Delta\sigma_0\phi^{1/2}[\exp(-\phi^{1/2}y) - \exp\phi^{1/2}(y-t)] \quad (11)$$

and so

$$F = 2bc\,\Delta\sigma_0\,[1+\exp(-\phi^{1/2}t) - 2\exp(-\phi^{1/2}t/2)]. \quad (12)$$

This result is obtained by substituting Equation 11 into Equation 2 and integrating between $t/2$ and 0.

To determine the value of $\Delta\sigma_0$ when the cracks occur we put $F = 2\sigma_{tu}dc$ into Equation 12 and hence

$$\Delta\sigma_0 = \sigma_{tu}\frac{d}{b}[1+\exp(-\phi^{1/2}t) - 2\exp(-\phi^{1/2}t/2)]^{-1}. \quad (13)$$

At this stage of the cracking sequence we have $t = s/2$ so a crack spacing of $s/4$ will result when

$$\Delta\sigma_0 = \sigma_{tu}\frac{d}{b}[1+\exp(-\phi^{1/2}s/2) - 2\exp(-\phi^{1/2}s/4)]^{-1}. \quad (14)$$

(d) Again cracks will occur mid-way between the present cracks and from the previous section a crack spacing of $s/8$ will result at

$$\Delta\sigma_0 = \sigma_{tu}\frac{d}{b}[1+\exp(-\phi^{1/2}s/4) - 2\exp(-\phi^{1/2}s/8)]^{-1}. \quad (15)$$

This cracking sequence will continue until the strength of the longitudinal plies is exceeded or debonding occurs if the shear stress in Equation 8 exceeds the shear strength of the interface.

Fig. 8 shows the theoretical crack spacing of a specimen of transverse-ply width of 3.2 mm and length 130 mm and demonstrates the stepped curve predicted from the theoretical analysis. The length of the steps in the curve increases as the overall applied stress increases; therefore, at high applied stresses the crack spacing can remain unchanged for a large range of applied stresses. The ultimate crack spacing of a given specimen will depend on its ultimate strength and, if its strength lies just below a value of applied stress at which another series of cracks should develop, an apparent limiting value of the crack spacing will be produced. Obviously if the ultimate strength of the specimen and the interfacial shear strength were infinite the crack spacing will tend to zero as the applied stress tends to infinity.

It is clear from the above analysis that the expected crack spacing will depend both on the position of the first crack and the specimen length.

Specimens of a given transverse-ply thickness and of length given by $s = 2nr$ where n is any integer and r is a given length, will have the same ultimate crack spacing if the first crack occurs half way along the specimen. The first crack will always occur at the same value of $\Delta\sigma_0$ but the second and third cracks (crack sequence b) will

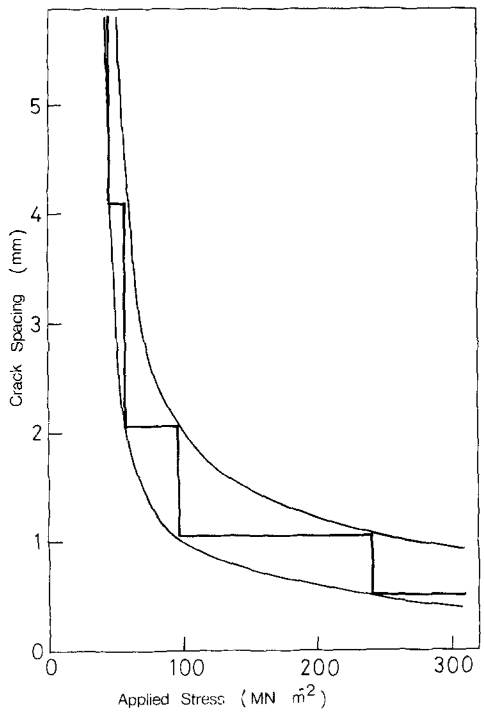

Figure 8 Theoretical crack spacing as a function of applied stress for an elastically bonded interface. The stepped curve shows the theoretical crack spacing when the first crack occurs half way down a specimen of length 130 mm and a transverse-ply thickness of 3.2 mm. The envelope drawn around the curve indicates the region in which the crack spacing is expected to fall for a specimen of any length and with a random position of the first crack.

occur at increasingly higher values of $\Delta\sigma_0$ for decreasing specimen length. In other words, the smaller specimens will tune into the stepped curve at a higher value of $\Delta\sigma_0$. If the specimen length lies between $2nr$ and $2(n+1)r$, similar stepped curves will be generated but they will be out of phase with each other. However, all the curves will lie within the envelope drawn around the stepped curve of Fig. 8. If the first crack does not occur at centre of the specimen but at a distance q from one end, the subsequent behaviour of the specimen will be similar to that of two specimens of length q and $(s-q)$. This means that one part of the specimen will have a different crack spacing from the other, unless $s/q = 2n$; however, the average crack spacing will also lie within the envelope of the stepped curve.

Thus, the theoretical crack spacing of a given specimen of any length and in which a random position of the first crack is taken will, for a given applied stress, lie within a fairly broad spread of values as indicated in Fig. 8.

4.2. Stress–strain curve

If transverse cracking occurs with a constant shear stress τ_i' acing at the interface, cracking will concontinue at a constant stress $E_1\,\epsilon_{tu}$, until the transverse ply has cracked into blocks of length between y' and $2y'$ after which the stress will rise with a modulus of $E_1 b/(b+d)$. The cracking will begin at a strain of ϵ_{tu} and the strain limit of cracking will be $\epsilon_{tu}E_c d/2E_1 b$ for crack spacings of $2y'$ and $3\epsilon_{tu}E_c d/4E_1 b$ for spacing of y'. However, if the interface is elastically bonded, the mean additional strain due to $1/l$ cracks per unit length produced as a result of an additional longitudinal-ply stress $\Delta\sigma_0$ is given by

$$\Delta\epsilon = \frac{2}{l}\int_0^{t/2} \frac{\Delta\sigma}{E_1}\,\mathrm{d}y \qquad (16)$$

or

$$\Delta\epsilon = \frac{2\Delta\sigma_0}{E_1\phi^{1/2}t}\,[1-\exp(-t\phi^{1/2}/2)]. \qquad (17)$$

As t becomes very small the modulus of the sample tends to $E_1 b/(b+d)$. The stress strain curve can be deduced from Equation 17 and the expected crack spacing at given applied stresses deduced earlier.

Fig. 9 shows the theoretical stress–strain curves deduced for the elastically bonded case for a given sample. In a similar manner to the theoretical behaviour of the crack spacing as a function of applied stress, the elastically bonded curve is stepped if a particular specimen length and position of the first crack is taken. If a random first crack position and a specimen of any length is considered, the theoretical stress–strain curve will lie within a fairly broad spread at strains higher the ϵ_{tu}. If the specimen is unloaded from a strain greater than ϵ_{tu}, the curve will return to the origin if the plies remain bonded. If debonding takes place a permanet set will develop only if load transfer is possible across the debonded surfaces. If no load transfer occurs the curve will also return to the origin.

5. Discussion

The straight-line plot of observed crack spacing against transverse-ply thickness in Fig. 5 at first indicated to us that a constant interfacial shear stress was acting between the plies. If we assume that the measured average crack spacing is equal to $3y'/2$, the gradient from Fig. 5, using Equation 3,

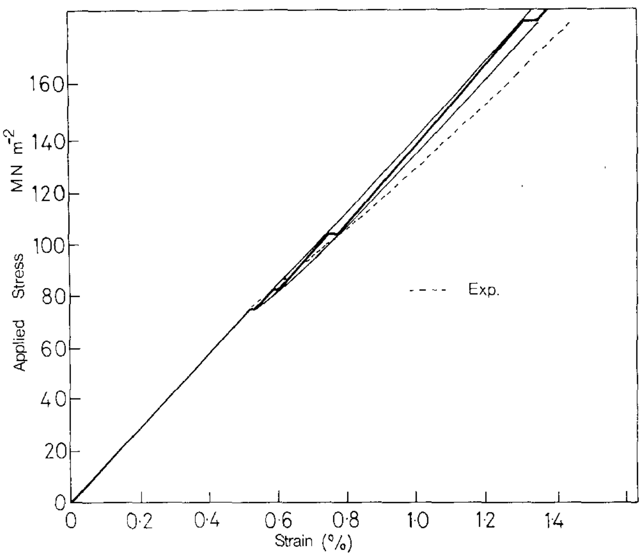

Figure 9 Theoretical stress–strain curve deduced for the elastically bonded interface with a transverse-ply thickness of 1.5 mm. The broken line indicates the experimental curve for the same specimen.

gives τ_i' equal to $8.6\,\mathrm{MN\,m^{-2}}$ (an average value of ϵ_{tu} calculated from the table has been used to arrive at this figure). The interlaminar shear strength of the ply interfaces was measured to be between 10 and $16\,\mathrm{MN\,m^{-2}}$ but the physical significance of the comparison between this value and τ_i' is not clear. The likelihood of a frictional bond between the plies giving a constant τ_i' formed for instance by a shear crack running up the interface seems remote, considering the structural integrity of the samples during testing. It is also unlikely that any debonded surfaces at the ply interfaces will be able to transfer any significant load. The experiments of Cooper and Sillwood, which showed multiple cracking in steel wire-reinforced epoxy composites, were performed by contracting the epoxy around the wire reinforcement by cooling the sample down to liquid nitrogen temperatures. As the expansion coefficient of epoxy is greater than that of steel, the steel epoxy interface is put under compression and under these conditions a load-carrying interface was formed.

No damage was observed at the interface between the longitudinal and transverse plies by microscope techniques. The bond strength between the plies was measured before and after the samples had been tensile tested and no significant difference was found. It is deduced from these experiments that the plies remained bonded throughout the tests.

Figs. 10 and 11 show the comparison between the experimental results of two representative sets of samples and the theory for the case when the plies are elastically bonded throughout. The observed crack spacing is always higher than that predicted by the theory for a given value of $\Delta\sigma_0$. Nevertheless, the theory supplies the general form of the experimental results. This is indicated in Fig. 5 where the measured ultimate crack spacing is compared with the theoretical predictions just before specimen failure. The observed effect of the increase in crack spacing with increasing ply thickness is demonstrated, but the slope of the curve is lower than that found experimentally.

However, there are a number of assumptions in the theory, the elimination of which would lead to a better correlation between theory and experiment. It has been assumed that the transverse ply has a unique breaking strain whereas in the cross-ply samples tested, a fairly wide distribution of strengths is to be expected along the transverse ply due to the random nature of imperfections, e.g. voidage and resin-rich areas and fibre bunching. The Kies [9] model of strain magnification in the transverse ply shows that interfibre spacing is very important in determining the magnification factors between fibres and hence the random nature of interfibre spacing, and fibre bunching will also produce a spread of transverse-ply strengths. The value of ϵ_{tu} used to determine the curves in Fig. 5 was that for the first crack. An average value of the failure strain of the transverse ply will be higher and a larger crack spacing than initially calculated will result. It has also been assumed that the interface behaves elastically and any inelastic behaviour of the resin will again tend to increase the crack spacing over that calculated for the elastic case.

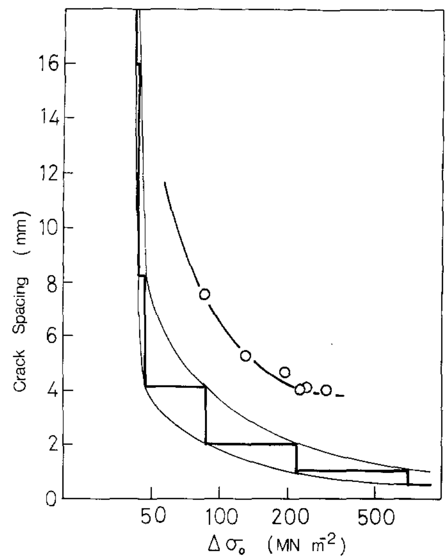

Figure 10 Average crack spacing as a function of applied stress. Comparison between the "elastically bonded interface" theory and experiment for a specimen with a transverse-ply thickness of 3.2 mm.

The maximum theoretical shear stress at the ply interface is given by Equation 8 when $y = 0$. Thus $\tau_{\text{max}} = b\Delta\sigma_0\phi^{1/2}$. Calculations indicate that τ_{max} will exceed the shear strength of the interface before specimen failure. Under these conditions the interface would be expected to debond until the interfacial shear stress falls below its shear strength. If the debonded part of the interface can transfer a significant stress, further decreases in crack spacing may result, but if the debonded surfaces cannot undergo stress transference, no further decrease in crack spacing is possible.

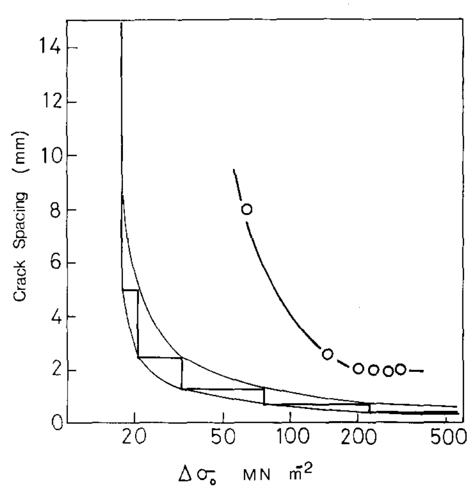

Figure 11 Average crack spacing as a function of applied stress. Comparison between the "elastically bonded interface" theory and experiment for a specimen with a transverse-ply thickness of 1.5 mm.

TABLE II

Transverse-ply thickness (mm)	Pre "knee" modulus (GN m^{-2})	Modulus just before failure (GN m^{-2})	
		Experimental	Theory
0.75	18.4	16.3	17.1
1.50	14.6	13.0	12.6
2.60	12.3	9.3	9.9
2.70	12.3	8.4	9.6
3.20	11.0	8.1	8.2

However, no debonding was observed experimentally, which indicates that Equation 8 overestimates the maximum shear stress at the crack surface, possibly because of the simple model used for the stress distribution although the discrepancy between τ_{max} and the interfacial shear strength will be reduced if a higher transverse ply failure strain is taken. A slower rate of stress transfer would also result in a larger theoretical crack spacing.

Fig. 9 demonstrates the comparison between the theoretical and experimental stress-strain curves for a typcial sample with a transverse ply thickness of 1.5 mm. Fair agreement is obtained for both the theoretical curves. Good correlation is obtained between the measured modulus just before failure and the theoretical modulus of $E_1b/(b+d)$ predicted for the stress–strain curves. This is indicated in Table II for the various specimens.

6. Conclusions

In this paper tensile experiments on cross-ply glass-reinforced plastics have been described. Multiple fracture has been found to occur in the transverse ply. It initiates at strains of between 0.4 and 0.5% and developed under a rising load in the sample. The spacing of these transverse cracks has been found to be dependent on the thickness of the transverse ply and the applied stress. Generally, the higher the applied stress and the smaller the transverse ply thickness, the smaller is the average crack spacing.

It has been shown empirically that the plies remain bonded throughout the multiple cracking and a simple theory has been advanced to predict the crack spacing for a given applied stress and transverse-ply thickness when the longitudinal and transverse plies are elastically bonded.

The theory indicates the general trend of the experimental results although it tends to underestimate the absolute value of the crack spacing. It should be emphasized, however, that because of the assumptions made in the analysis, an underestimate of the crack spacing is expected, but we believe that the theoretical analysis will be a useful aid in exploring the transverse-damage phenomenon in cross-ply laminates.

Appendix

A modified shear lag analysis [23] is used to determine the stress in the longitudinal after the first crack has occurred in the transverse ply. We assume there is elastic continuity between the plies of the cross-ply composite shown in Fig. 1 and that the tensile strains in the longitudinal and transverse plies are equal at large values of y.

The first crack in the transverse ply occurs at a strain ϵ_{tu} and at this point the stress in the longitudinal ply will be $E_1\epsilon_{tu}$ and so the additional stress $\Delta\sigma$ at a distance y from the crack will be

$$\Delta\sigma = \sigma_1 - E_1\epsilon_{tu}, \tag{A1}$$

where σ_1 is the stress in the longitudinal ply at a distance y from the crack. It is assumed that

$$\frac{d\Delta\sigma}{dy} = H(u-v), \tag{A2}$$

where H is a constant and u and v are the y-components of elastic displacement in the longitudinal and transverse plies respectively. Therefore,

$$\frac{d^2\Delta\sigma}{dy^2} = H\left(\frac{\sigma_1}{E_1} - \frac{dv}{dy}\right). \tag{A3}$$

We also assume that

$$\frac{dv}{dy} = \frac{d\bar{v}}{dy} = \bar{\epsilon}_t,$$

where $\bar{\epsilon}_t$ is average strain in the transverse ply. We have

$$\frac{d\bar{v}}{dy} = \bar{\epsilon}_t = \frac{1}{E_t d}\left[E_c\epsilon_{tu}(b+d) - \sigma_1 b\right] \tag{A4}$$

because the tensile load supported by all cross-sections of the composite must be the same.

Substituting Equation A4 into Equation A3 we have, using Equation A1,

$$\frac{d^2\Delta\sigma}{dy^2} = \phi\Delta\sigma \tag{A5}$$

where

$$\phi = \frac{HE_c(b+d)}{E_1E_td} \tag{A6}$$

The general solution of Equation A6 subject to the conditions $\Delta\sigma = 0$ at large y and $\Delta\sigma = \Delta\sigma_0$ at $y = 0$ is

$$\Delta\sigma = \Delta\sigma_0 \exp(-\phi^{1/2}y). \tag{A7}$$

To find H we assume that the condition of stress equilibrium is given by

$$\frac{\partial\tau_{xy}}{\partial x} = 0. \tag{A8}$$

Hence $\tau_{xy} = \text{constant} = \tau_i$ where τ_{xy} is the shear stress on planes perpendicular to the interface and τ_i is the shear stress at the interface.

Let w be the actual displacement of the transverse ply close to the longitudinal ply. At the interface $w = u$, and at large x, $w = v$. Thus we have

$$\frac{dw}{dx} = \frac{\tau_x}{G_t} = \frac{\tau_i}{G_t}, \tag{A9}$$

where G_t is the shear modulus of the transverse ply in the y direction.

Integrating Equation A9 we have

$$\Delta w = v - u = \frac{\tau_i d}{G_t}. \tag{A10}$$

Hence

$$H = \frac{G_t}{bd}. \tag{A11}$$

This result is obtained by substituting Equations 7 and A10 into Equation A2. So

$$\phi = \frac{E_cG_t}{E_1E_t}\frac{b+d}{bd^2} \tag{A12}$$

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